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HELIUM THREE

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ABSTRACT

A review is given of the present state of knowledge concerning the condensed phases of He^3. Attention is confined to the pure substance, and emphasis is placed upon the theoretical understanding of the material.

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I. INTRODUCTION

A. Outline

In presenting this "keynote" lecture on He^3, I obviously can not --and should not--give an exhaustive summary of the present state of knowledge of the subject. I propose to spend the available time looking at the field from a broad point of view, with emphasis on what is understood and understandable and little mention given to those more complicated parts of the subject which are still primarily unrelated experimental results. I will thus concentrate on a "theoretical approach," employing the experiments in a corroborative manner, although it should be realized that this is not an historically accurate presentation.

To remove, somewhat, the bias of this approach I include references to a number of review papers,[1-5] and will refer at the appropriate junctures to the recent original literature.

Unfortunately, because of lack of time, I must concentrate on the pure substance and not comment at all on the interesting properties of mixtures (both solid and liquid) of He^3 and He^4, or upon ions and impurities in He^3.

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B. Interest in He^3

I think it is appropriate to ask--not too frequently, but perhaps at a time like this--why we are interested in He^3. Certainly, we can not claim that research on He^3 is on the front line of fundamental research. That is, we know the basic laws of physics governing He^3, and we can't reasonably expect to discover in the study of He^3 any modifications in the principles of quantum mechanics. But that, I would suggest, does not make it an uninteresting subject, for the basic laws may be vacuous--or almost so--in a complicated problem. To understand a complicated situation we must develop new laws--usually called "approximations"--which in principle are derivable from the fundamental laws, but in practice are not so derived. The investigation of complicated systems for the purpose of discovering such laws is, I believe, as fundamental as research on the "front line of physics".

Of the complicated systems--namely many-body systems--there are some which are sufficiently simple that we now believe that we understand them very well. I think, for example, of a photon gas or a classical gas such as air under normal conditions. The simplest many-body system that we do not entirely understand is an isotropic, homogeneous, collection of a single species of fermions that have no charge and spin 1/2, and interact with central forces. (I choose fermions because this seems a priori simpler in not having a Bose condensation, so perturbation methods should be valid, but this argument may be specious. Certainly a corresponding system of bosons is of equal fundamental importance.) Now, of course, I have just described He^3 and, except for the complicating fact that the interatomic forces are strong rather than weak, one can see that it plays a fundamental role in the physics of many-body systems. I think,

that this is basically the reason we are interested in He^3.

Unfortunately, our theoretical understanding of even so simple a many-body problem as He^3 is essentially limited to the very-low-temperature region below 0.1° K (I am, of course, excluding the high-temperature classical range), and so--in line with the approach of this lecture--I shall summarily exclude from consideration almost all properties of the substance above 1°K. Although the high-temperature range is beyond our comprehension at present, we shall see that there is plenty below 0.1°K to interest both the theoretical and the experimental physicist. In fact, the subject should prove exciting for many years to come.

II. THE FERMI LIQUID, He^3

In 1956, Landau[6] proposed a phenomenological theory of a strongly interacting system of fermions at very low temperatures. The most significant development in research on He^3 since the Low Temperature Conference of 1960 has been the complete acceptance of Landau's theory by (thinking) physicists. On the one hand, the experimentalists have confirmed the predictions of the theory for all quantities, both measured and computed (and only a few remain yet to be tested, as we shall see), while on the other hand, the theorists have succeeded in supplying the theory with a rigorous underfooting by starting from microscopic theory and making (only a few) well-defined hypotheses.

Although Landau's work may be familiar to most of you, in view of its significance, I will review his theory in the next section. Subsequently it will be compared with experiment, after which zero sound will be discussed. We will end the discussion of the liquid with comments upon various microscopic theories.

A. Landau's Phenomenological Theory

Landau's theory is based on the assumed validity of perturbation theory starting from a gas of noninteracting atoms. By perturbation theory one does not mean just a low-order calculation, but simply that the theory converges. Proofs of this have been given by Bloch and de Dominicis,[7] Klein,[8] Nozieres and Luttinger,[9] and other workers. Later in the lecture we will explore the possible consequences of nonconvergence, but it should be realized that, except possibly at low temperatures, the validity of perturbation theory is well supported by experiment.

A consequence of this assumption is that the elementary excitations of the true system can be related to the properties of the noninteracting system. It thus follows that the elementary excitations--or quasi-particles--are equal in number to the number of He^3 atoms, obey Fermi statistics, and can be characterized by a momentum quantum number. They may be thought of as atoms surrounded by their polarization field. (It should be noted that there may be excitations of the real system which can not be realted to the unperturbed system, because they disappear when the interaction is reduced to zero. Such an excitation may still be considered in the Landau theory, as a collective interaction between quasi-particles. An example is zero sound.) Actually the concept of a quasi-particle is common to the theory of many complicated systems. One thinks, for example, of phonons and rotons in liquid He^4, phonons and Bloch waves in a solid, and the shell model and optical model of nuclear physics.

It is realized that in a many-body system the elementary excitations will not be stable in the sense that a quasi-particle will excite other

quasi-particles and thus degrade its energy by the creation of many excitations. For a description of the system in terms of quasi-particles to be valid, we must demand that the quasi-particles have long lifetimes. That is, the energy of a quasi-particle $\epsilon(p)$ must be much larger than $\hbar/\tau$, where τ is the life time of the excitation. The lifetime is dominated (at low temperatures) by the exclusion principle, and $1/\tau$ varies as the square of the distance from the Fermi surface (in momentum units). For He^3, we find well-defined quasi-particles assocaited with thermal excitations having temperatures less than 0.05° K.

The above ideas may be formalized by letting $n(\underset{\sim}{p}, \underset{\sim}{\sigma})$ be the distribution function for quasi-particles of momentum $\underset{\sim}{p}$ and spin orientation $\underset{\sim}{\sigma}$. If E is the total energy of N atoms of mass m in a volume V, then we may define the energy of a quasi-particle $\epsilon(\underset{\sim}{p}, \underset{\sim}{\sigma})$ by the variation in the total energy upon adding one quasi-particle, that is

$$\delta E = \sum_{\underset{\sim}{\sigma}} \int \epsilon(\underset{\sim}{p}, \underset{\sim}{\sigma})\, \delta n(\underset{\sim}{p}, \underset{\sim}{\sigma})\, \frac{d^3p}{(2\pi\hbar)^3} \quad . \tag{1}$$

Similarly, we may ask how one quasi-particle influences the energy of another, thus defining the basic function of the Landau theory $f(\underset{\sim}{p}, \underset{\sim}{p}', \underset{\sim}{\sigma}, \underset{\sim}{\sigma}')$ by the relation

$$\delta\epsilon(\underset{\sim}{p}, \underset{\sim}{\sigma}) = \sum_{\underset{\sim}{\sigma}'} \int f(\underset{\sim}{p}, \underset{\sim}{p}', \underset{\sim}{\sigma}, \underset{\sim}{\sigma}')\, \delta n(\underset{\sim}{p}', \underset{\sim}{\sigma}')\, \frac{d^3p'}{(2\pi\hbar)^3} \quad . \tag{2}$$

In an ideal gas, f is zero, but in the theory of a Fermi liquid, f plays a vital role. It is considered to be determined phenomenologically, although it can be derived from any microscopic theory of the liquid.

It is now very easy to show that the quasi-particle distribution function is just[6]

$$n(\underset{\sim}{p}, \underset{\sim}{\sigma}) = \left[e^{(\epsilon-\mu)/kT} + 1 \right]^{-1} , \tag{3}$$

where the chemical potential μ is determined (as usual) by the normalization criterion

$$N = V \sum_{\underset{\sim}{\sigma}} \int n(\underset{\sim}{p}, \underset{\sim}{\sigma}) \frac{d^3p}{(2\pi\hbar)^3} . \tag{4}$$

It is convenient to define an effective mass (at the Fermi surface) by:

$$\nabla_{\underset{\sim}{p}} \epsilon(\underset{\sim}{p}, \underset{\sim}{\sigma}) \Big|_{p=p_F} = \frac{p_F}{m^*} , \tag{5}$$

where the Fermi momentum is

$$p_F = \hbar \left(\frac{3\pi^2 N}{mV} \right)^{1/3} . \tag{6}$$

The effective mass plays an important role in the theory since at low temperature only the region near the Fermi surface is of importance. (In fact, it is only in this region that the quasi-particles are well-defined.) In the same manner, we may see that f will only be needed for its arguments near the Fermi surface, so we write

$$f(\theta,\ \underset{\sim}{\sigma}, \underset{\sim}{\sigma}') = f(\underset{\sim}{p}, \underset{\sim}{p}',\ \underset{\sim}{\sigma}, \underset{\sim}{\sigma}')\Big|_{p,\,p'=p_F} = \frac{(2\pi\hbar)^3}{8\pi p_F m^*} \sum_{\ell} (F_\ell + \underset{\sim}{\sigma}\cdot\underset{\sim}{\sigma}'\ Z_\ell)\ P_\ell(\cos\theta)\ , \tag{7}$$

where invariance arguments have been employed to fix the spin dependence, θ is the angle between $\underset{\sim}{p}$ and $\underset{\sim}{p}'$, and the constants F_ℓ and Z_ℓ are to be determined by experiment. (The notation is that of Hone.[10])

With this formalism Landau derived a relation between m* and f (from Galilean invariance):

$$\frac{\underset{\sim}{p}}{m} = \underset{\sim}{\nabla}_p\, \epsilon(p,\sigma) - \sum_{\underset{\sim}{\sigma}'} \int f(\underset{\sim}{p}, \underset{\sim}{p}', \underset{\sim}{\sigma}, \underset{\sim}{\sigma}')\ \underset{\sim}{\nabla}_{p'} n(\underset{\sim}{p}', \underset{\sim}{\sigma}')\ \frac{d^3p'}{(2\pi\hbar)^3}\ , \tag{8}$$

which may be written as

$$F_1 = 3\left(\frac{m^*}{m} - 1\right)\ . \tag{9}$$

Landau also showed that in the limit of low temperature the specific heat C was related to that of the ideal gas C_F by

$$\frac{C}{C_F} = \frac{m^*}{m}\ , \tag{10}$$

(and thus was predicted to have a linear dependence on the temperature), while the velocity of sound S was given by

$$S = \frac{p_F}{\sqrt{3}m}\left[\frac{1 + F_0}{1 + \frac{F_1}{3}}\right]^{1/2}\ , \tag{11}$$

and the susceptibility χ was related to that of the ideal gas χ_F by

$$\frac{\chi}{\chi_F} = \left[\frac{1}{1 + \frac{F_1^s}{3}} \right] \left[\frac{1}{1 + \frac{Z_0}{4}} \right], \tag{12}$$

(and thus approaches a constant at low temperatures).

The transport properties of the liquid have been considered by Landau,[6] Khalatnikov and Abrikosov,[11] and Hone.[12] The calculations proceed as in kinetic theory, and the result involves a quasi-particle--quasi-particle scattering function $w(\theta, \phi)$. The Fermi liquid behavior does not appear in the viscosity κ, but does appear explicitly in the spin self-diffusion coefficient D. As can readily be understood in terms of the exclusion-principle effect on quasi-particle scattering, it is predicted that for low temperatures η and D vary as T^{-2}, and κ varies as T^{-1}.

Landau[13] has suggested approximating the scattering function $w(\theta, \phi)$ by the forward-scattering amplitude $a(\theta, \underset{\sim}{\sigma}, \underset{\sim}{\sigma}')$ for two quasi-particles colliding at initial angle θ, which can be written

$$w(\theta, \phi) \approx \frac{2\pi}{h} \left[a^2(\theta, \underset{\sim}{\sigma}, \underset{\sim}{\sigma}') \right]_{av.} . \tag{13}$$

Now interestingly enough, the quantity $a(\theta, \underset{\sim}{\sigma}, \underset{\sim}{\sigma}')$ may be related to $f(\theta, \underset{\sim}{\sigma}, \underset{\sim}{\sigma}')$. However, it is not equal to f since in a we consider a scattering with no energy change but some (small) momentum transfer, while in f the quasi-particles travel strictly forward but scatter with some (small) energy transfer. Landau[13] has shown how to relate f and a, and thus allowed numerical estimate of η, κ, and D.

B. Comparison With Experiment

A comparison of the Landau theory with experiment has been given in a recent paper by Hone,[10] so we limit ourselves to only a brief summary of the results at the saturated vapor pressure, where from the observed density one concludes that $p_F/h = 0.785 \times 10^8$ cm^{-1}. The velocity of sound, low-temperature specific heat, and susceptibility all have the appropriate temperature dependence and imply that $m^*/m = 2.82$, $F_0 = 9.25$, $F_1 = 5.46$, and $Z_0 = -2.80$. The other parameters of the theory have not yet been evaluated. Note the rather large value of m^*, while the value of $Z_0 = -0.7$ shows via Eq. (12) that the system is very close to being ferromagnetic. This large spin-spin interaction comes from the exclusion principle of course and is not a magnetic spin-spin effect. It is an important aspect of the behavior of the liquid (and the solid).

From these experimentally determined parameters, using the theory one can predict that as the temperature goes to zero, $\eta T^2 = 1.5 \times 10^{-6}$, $\kappa T = 57$, and $DT^2 = 4.2 \times 10^{-6}$ (in cgs units). Recent experiments confirm the predicted temperature behavior (which should be contrasted with the situation presented at the Low Temperature Conference of 1960[4]), and yield $\eta T^2 = 2.8 \times 10^{-6}$, $\kappa T = 48 \pm 3$, and $DT^2 = 1.54 \times 10^{-6}$. The numerical agreement should be considered good, in view of the approximate theoretical work. In fact, the attempt to relate $w(\theta, \phi)$ to f should probably be abandoned and the experiments used simply to determine w. Alternatively, perhaps microscopic theory can be employed to shed light on the ϕ dependence of w and so improve the theoretical estimates.

It should be noted that the viscosity measurement (below 0.1° K) is not a direct measurement, but is deduced from the absorption of sound and thus involves additional theory. A direct measurement would be most

welcome, as would measurements of the density dependence of η and κ in the Fermi liquid region. This latter measurement would afford additional checks on the theory, because the equilibrium properties are known, and κ and η have already been estimated as functions of density.[10]

The experimental situation with regard to the spin-spin relaxation time T_2 is still not resolved,[1] but the spin-lattice relaxation time T_1 has been measured recently down to a relatively low temperature of 0.4° K (Communication 1.21). At these temperatures it is starting to display departures from the classical theory. It will be most interesting to obtain experimental results at lower temperatures, although T_1 and T_2 remain to be calculated for a Fermi liquid.

C. Zero Sound

As the temperature approaches zero, ordinary sound in liquid He^3 is strongly attenuated; but Landau has pointed out that a new possibility exists in a Fermi liquid, namely the oscillation of the self-consistent potential in which quasi-particles move.[14] This collective motion is closely related to plasma oscillations, and to the vibrational and rotational states of atomic nuclei.

For propagation of ordinary sound local thermal equilibrium must be maintained; while zero sound requires the opposite, criterion, namely that the mean free path of quasi-particles be large compared to the wavelength of the collective oscillation. Numerically this requires, for example, that at 1000 Mc/sec the temperature must be below 0.05° K to propagate zero sound. For six years this has discouraged the experimentalists, but a serious attempt is now being made to observe it (Communication 1.14).

The mathematics required to derive zero sound is rather simple. One allows for spatial inhomogeneity in the Fermi liquid theory, and

treats the variation from equilibrium to first order. Thus, we have

$$n(\underset{\sim}{p}, \underset{\sim}{r}, \underset{\sim}{\sigma}, t) = n_0(\underset{\sim}{p}, \underset{\sim}{\sigma}) + \delta n(\underset{\sim}{p}, \underset{\sim}{r}, \underset{\sim}{\sigma}, t)$$

$$\epsilon(\underset{\sim}{p}, \underset{\sim}{r}, \underset{\sim}{\sigma}, t) = \epsilon_0(\underset{\sim}{p}, \underset{\sim}{\sigma}) + \delta\epsilon(\underset{\sim}{p}, \underset{\sim}{r}, \underset{\sim}{\sigma}, t) \quad . \tag{14}$$

Now Eq. (2) becomes

$$\delta\epsilon(\underset{\sim}{p}, \underset{\sim}{r}, \underset{\sim}{\sigma}, t) = \sum_{\sigma'} \int f(\underset{\sim}{p}, \underset{\sim}{p}', \underset{\sim}{\sigma}, \underset{\sim}{\sigma}') \, \delta n(\underset{\sim}{p}', \underset{\sim}{r}, \underset{\sim}{\sigma}', t) \, \frac{d^3p'}{(2\pi\hbar)^3} \quad . \tag{15}$$

Since at low temperatures the quasi-particle mean free path is very long, we may approximate the Boltzmann equation by ignoring the collision term. Thus we have

$$\frac{\partial n}{\partial t} + \underset{\sim}{\nabla}_r n - \underset{\sim}{\nabla}_p \epsilon \cdot \underset{\sim}{\nabla}_p n \cdot \underset{\sim}{\nabla}_r \epsilon = 0 \, , \tag{16}$$

which can be linearized to

$$\frac{\partial(\delta n)}{\partial t} + \underset{\sim}{\nabla}_r(\delta n) \cdot \underset{\sim}{\nabla}_p \epsilon_0 - \underset{\sim}{\nabla}_r(\delta\epsilon) \cdot \underset{\sim}{\nabla}_p n_p = 0 \, . \tag{17}$$

Clearly, this equation has solutions in which $\delta\epsilon$ and δn vary harmonically in space and time. Inserting Eq. (15), and the properties of n_0 and ϵ_0, one can find the dispersion relation for these waves.[11] Depending upon the parameters, the waves can either propagate without damping (in this approximation of ignoring the collision term), propagate with damping, or propagate with ever-increasing amplitude. The last would indicate that the Fermi liquid theory is inappropriate for describing the system;

numerically one finds that He^3 corresponds to the first case, that is, the attenuation of zero sound decreasing as the temperature decreases. This result refers only to the simplest mode of density fluctuation. More complicated density modes, and all spin-dependent modes, seem to be damped; although there appears to be the possibility of propagating a spin-dependent mode in the presence of an external field.[11]

D. Microscopic Theories

The most naive theory of liquid He^3 would be that of an ideal Fermi gas. As we have seen above, this can not be very good, because the function $f(\underset{\sim}{p}, \underset{\sim}{p}', \underset{\sim}{\sigma}, \underset{\sim}{\sigma}')$ (which is zero for an ideal gas) plays a vital role in the description of He^3. A Hartree-Fock theory suffers from the fact that the interatomic potential is strongly singular at short distances so that two-body correlations must be included to obtain even finite results.

Considerable study has been made of a hard-sphere gas in the low-density limit.[11] This work is of formal interest, but may not be applied directly to He^3, which can not be considered dilute in that $r_0(N/V)^{1/3} > 1$ (where r_0 is the range of the forces between atoms), while the reverse inequality is necessary for the validity of the theory.

The only serious attempt to calculate the properties of He^3 from first principles is due to Brueckner and Gammel.[15] This calculation is now four years old and has been frequently discussed, so that I will only comment briefly upon it. Proceeding somewhat intuitively, they calculate the interaction between two atoms carefully, while taking into account the rest of the medium in only two regards. The first is through the effect of the exclusion principle which limits the states available for the two atoms under consideration, and the second is through a self-consistent potential in which the two atoms move. It can be seen

that the theory has many of the features of Landau's theory, and can be considered as a special case of the Fermi liquid. The numerical results are rather reasonable (but only good to about a factor of two) and can perhaps be interpreted as suggesting that two-body correlations are the most important in describing the liquid.

In the past few years many of the powerful techniques of formal quantum field theory have been applied to the quantum statistical mechanics of the many-body problem, notably by Galitskii and Migdal,[16] Luttinger and Ward,[17] Matsubara,[18] and Martin and Schwinger.[19] This work has led me to a derivation of the Landau theory,[7,8,9] showing that it follows from a microscopic theory in which the true system is related to the system of noninteracting particles by means of a convergent perturbation theory. This is a most important result, and is especially welcome in that it allows a clear and precise definition of a quasi-particle. One can describe the present situation by saying that we know what a quasi-particle is, but we do not yet know how to calculate its properties from the properties of free He^3 atoms. In a sense, doing the hard calculation to relate a quasi-particle to a particle is not so important as knowing that there is such a relation and that the quasi-particle is a well-defined concept.

Nevertheless, the formal techniques do suggest a number of new approximation methods. In particular, one can formulate a consistent two-body correlation approximation to the liquid. Such a method has been applied to nuclear matter by Puff,[20] and is now being applied (and extended) to liquid He^3 by R. E. Mills[21] and D. Beck.[22] I think it will be most exciting to see if such a (conceptually) simple approximation yields a quantitative description of the fluid.

Another interesting consequence of the formal work is the possibility of extending the Landau theory to higher temperatures. The microscopic theory is not limited to the range in which quasi-particles are a well-defined concept, and a study of the theory may suggest new phenomenological descriptions valid at higher temperatures. Mr. C. Sung has undertaken such a study, in an attempt to predict the high-temperature deviation of the transport coefficients from Fermi liquid behavior (in terms of the measured equilibrium properties).[23] An attempt in this direction, but rather different in approach, has recently been made by Nishimura and Mori.[24]

I would like to call attention to a rather original approach recently put forward by Wu and Feenberg.[25] First they relate He^3 to a boson system of particles of the same mass, and preliminary calculations seem to indicate that, given the properties of the boson system, they have a rapidly convergent approximation for the fermion system. They then suggest a two-body correlative approximation to the boson system, which hypothesis they can check against experiments on liquid He^4. If they are successful in this test, they will repeat the computation for bosons of mass 3 and thus have a theory of the ground state of liquid He^3. The method seems more suited to computing ground-state properties than the properties of the low-lying excitations.

III. SOLID He^3

The experimental information concerning the solid is not nearly as complete as that about the liquid. Although qualitatively no doubt correct, the theories of the solid are still in need of quantitative improvements. In view of this, and in the spirit of this lecture, I will comment now on the general features of the solid without going into

any of the (interesting) details. In future conferences we shall surely devote more attention to the solid phase.

A. Spin Ordering

Liquid He^3 is particularly interesting because it is a "quantum" liquid, and the same comment may be applied to the solid phase. Other solids than He^3 and He^4 can be well understood by treating the dynamics of the atoms with classical theory, but He^3 can not, because the zero-point motion causes the rms deviation of an atom from its lattice site to be about 30% of the interparticle separation at the melting curve.[1] As a consequence, the solid properties are sensitive to the symmetry of the nuclear wave function and thus an effective coupling is created between the nuclear spins, as was first observed by Prinakoff,[26] and subsequently estimated by Bernardes and Primakoff.[27]

The coupling between spins is not as large in the solid as in the liquid, since the wave functions for atoms overlap much more in the liquid. Thus as the temperature is lowered from the classical range in which the entropy of the liquid, S_ℓ, is larger than the entropy of the solid, S_s, one finds that the entropy of the solid becomes $R \ln 2$ (to about 1%) below 0.5° K, and soon one reaches a temperature at which the melting curve has a minimum and $S_\ell = S_s$. This equality has been quantitatively confirmed at the melting-curve minimum temperature $T_m = 0.33^\circ$ K (Communication 1.7). As the temperature decreases further the solid spins align and thus S_s drops to zero as T^3. In this region, S_ℓ varies as T, and therefore S_s eventually equals and then becomes less than S_ℓ. The predicted maximum in the melting curve is difficult to estimate quantitatively, but the most careful theoretical treatment of Saunders predicts a temperature of about 0.05° K.[28] The maximum has yet to be observed, but studies of the

susceptibility indicate that it is being approached,[1] and give an "extrapolated" temperature in agreement with theory.

B. Crystallographic Phases

The solid is observed in a bcc (α) phase just above the melting curve, then converts to an hcp (β) phase at pressures of the order of 100 kg/cm^2. At very high pressures, and above 18° K, it converts to an fcc lattice.

The behavior near the melting curve has recently been clarified by H. Fairbank,[29] following some experimental confusion. The conversion from α to β phase has been studied by Bernardes, who has shown that the relative stability is a very subtle question, but that the experimental data on thermal conductivity and specific heat are both consistent with each other and with a simple crystallographic transition (and not a magnetic transition as was previously speculated).[30] The theory of Saunders does predict the α phase as the more stable phase near the melting curve, and furthermore makes interesting predictions concerning the magnetic properties under pressure. In particular, it is shown that at high densities where spin ordering occurs at relatively high temperatures, the nearest-neighbor approximation fails and next-nearest neighbors must be simultaneously considered (which is unfortunately beyond the present theory).[28] The theory of Bernardes[1] is in good agreement with the measured Grueneisen constants for both the α and β phases. Saunders has successfully used the spin relaxation times in the α phase as a test of his theory. An anomaly in the specific heat of the α phase[31] has yet to be explained,[31] but is presumably a consequence of the bcc structure.

The hcp-to-fcc transition has not been examined theoretically, but presumably is quite classical in nature because it occurs under approximately the same conditions in solid He^4.

IV. POSSIBLE SUPERFLUID PHASE

A low-temperature superfluid phase of liquid He^3 has yet to be observed experimentally, but there are compelling reasons for believing that such a phase exists. In this section I will review the present status of theoretical considerations concerning the existence of such a possible phase. Properties of the new phase have been reviewed elsewhere.[1,3]

A. Breakdown of Perturbation Theory

As was discussed above, the formal theories of quantum statistical mechanics have shown that the Landau theory is valid if the real system of liquid He^3 may be related to a system of noninteracting particles by perturbation theory. This allows the formulation of a clearly defined question, namely, is there a temperature T_c below which perturbation theory is invalid? It should be emphasized that this is a very precise concept. It may be difficult to answer the question, but there can be no doubt as to the meaning of the question.

This question was first asked in 1958, when it was observed that the BCS theory of superconductivity implied a ground state that could not be obtained from that of a noninteracting gas by perturbation theory, and furthermore, that the theory could be generalized to apply to any system of interacting fermions.[32] In 1959 it occurred independently to two groups in the West[33] and to Landau and Pomeranchuk[34] that the necessary and sufficient criterion for a BCS-type ground state is an attractive interaction between quasi-particles in any relative angular-momentum state. On this basis computations have been made; this work has been

reviewed elsewhere,[2,3] and I will discuss only the results (in the next section).

Returning to the general formulation of the problem, one can see that the work employing BCS theory exploits only one possible way in which perturbation theory can fail. Furthermore, the approach is to construct a theory of a state which has never been observed, and then imagine the temperature being raised and ask when the theory breaks down. It would be much more appealing to examine the theory of the normal liquid, and find a temperature below which the theory is not valid. Such an approach could employ the many experimental results on the normal fluid, and in addition be repeatedly checked against the properties of the observed phase.

Now in liquid He^3 we expect two-body correlations to be dominant, so we are led to examine the two-body interactions in the normal fluid to see if they ever become strong enough to cause a correlation function that can not be obtained by perturbative methods. Thouless was the first to do this, and he showed that an elementary estimate of the temperature below which perturbation theory failed yielded the same temperature as that above which BCS theory is no longer valid.[35] This calculation opens the way for more-accurate estimates of the transition temperature for liquid He^3.

Now I wish to emphasize that predicting the transition temperature is extremely difficult. The transition temperature depends exponentially on the effective interaction between quasi-particles, and it will be noted that even in the theory of superconductivity, the transition temperature is not computed, but rather taken from experiment. Furthermore, it is very difficult to calculate the effective interaction between quasi-particles

with any precision. We do not doubt the existence of quasi-particles or that they surely attract each other in high-angular-momentum states, but it is quite something else to compute this interaction with accuracy.

What is needed, is a quantitatively accurate microscopic theory of the normal fluid. Having this, there is no difficulty in finding a temperature below which the theory doesn't converge, and if the theory gives an accurate description of the normal fluid (as can be checked by comparison with experiment) then the predicted transition should be accurate. It is largely for this reason that I am so anxious to see if the two-body correlation approximation to the normal liquid is successful. In general, the propagation of a quasi-particle of momentum $\underset{\sim}{p}$ may be described by a Greens function $G_>(\underset{\sim}{p},t)$ which has the spectral representation

$$G_>(\underset{\sim}{p},t) = \int_{-\infty}^{\infty} \frac{d\omega\, A(\underset{\sim}{p},\omega) e^{-i\omega t}}{2\pi i \left[1 + e^{-(\omega-\mu)/kT}\right]} \tag{18}$$

where μ is the chemical potential, and the spectral function $A(\underset{\sim}{p},\omega)$ describes all the thermodynamic properties of the normal fluid. In the two-body-correlation approximation, the transition temperature T_c is given by the highest temperature at which there is a nontrivial solution to the equation

$$\Psi(\underset{\sim}{p}') = \frac{1}{2}\int \frac{d^3p'}{(2\pi)^3}\; v(\underset{\sim}{p}-\underset{\sim}{p}')\,\Psi(\underset{\sim}{p}') \int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}$$

$$\frac{A(\underset{\sim}{p},\omega)\,A(-\underset{\sim}{p}',\omega)}{(2\mu-\omega-\omega')}\left[\tanh\left(\frac{\omega-\mu}{2kT_c}\right)+\tanh\left(\frac{\omega'-\mu}{2kT_c}\right)\right]. \tag{19}$$

In this equation, $v(\underset{\sim}{p}-\underset{\sim}{p}')$ is a Fourier component of the interatomic potential. If we approximate the spectral function by

$$A(\underset{\sim}{p},\ \omega) = 2\pi\delta(\omega - \epsilon(\underset{\sim}{p}))\ , \tag{20}$$

then Eq. (19) reduces to the BCS equation, which has been previously employed to estimate T_c. A more accurate estimate of T_c must await a better spectral function and, of course, the accuracy of the spectral function is subject to check by computing the properties of the normal system.

Recently it has been proposed by Gorkov and Pitaevskii that the two-body approximation may not suffice for two quasi-particles interacting in high-angular-momentum states.[36] In this case, the quasi-particles are at rather large relative separations and the intervening fluid reduces the net force between the particles. They show that at large distances the interatomic force is reduced by a factor

$$\frac{m s_0^2}{m^* s^2} \approx 1.7 \times 10^{-2}\ , \tag{21}$$

where s and s_0 are the velocities of sound in liquid He^3 in an ideal Fermi gas, respectively. If this factor were in effect at all ranges (and it clearly isn't at very small distances), then it would reduce T_c significantly. Unfortunately, it is not yet known how important is shielding for those distances involved with pairing in the (not-so-large) $\ell = 2$ state.

B. The Transition Temperature

The first quantitative estimates of T_c yielded values of $T_c \approx 0.08^\circ$ K.[33] Subsequently Bardasis and Schrieffer[37] and Morel and

Noziores[38] computed the effect of finite quasi-particle lifetime, which is essentially the inclusion of some of the effect of not replacing the spectral function $A(\underset{\sim}{p},\omega)$ with a delta function. They find that $T_c \approx 0.02^\circ$ K. Gor'kov and Pitaevskii have extrapolated their result for condensation in states of large angular momentum ℓ, to the case where $\ell = 2$. They thus find a lower bound to $T_c \approx 2 \times 10^{-4}\ ^\circ$K. (This does not include quasi-particle damping, which could lower T_c by another factor of two or three.) On the other hand, Gor'kov and Pitaevskii argue that it is inconsistent to use an effective mass m^* (the effect of the medium on a quasi-particle propagator) and not include the renormalization factor on the potential. This argument is suggestive, but certainly not compelling. On this basis they propose a second estimate (again ignoring the damping factor) as that obtained with $m^* = m$; namely $T_c \approx 8 \times 10^{-3}\ ^\circ$K.[39]

The experimentalists must find it discouraging to have the theorists continually lowering T_c. I have tried to make it clear that the theory is not now capable of quantitatively evaluating T_c, but--and I believe this has been the spirit in which the theoretical estimates have been made--the best calculations give some basis for hoping that T_c is within an experimentally attainable range, and thus should serve to encourage the experimentalists to even more heroic efforts.

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